

Assignment 3: Atomic physics applications

Compiled: August 3, 2026

Learning outcomes and objectives

This assignment will give you practice in

- The computation of expectation values for a two-particle wavefunction
- Evaluating integrals to perform perturbation theory calculations
- Calculations relevant for the emission and absorption of radiation
- Finding density matrix representations

Exercise 1 *Stern-Gerlach: measurement edition* (8 points)

Consider an ensemble of silver atoms incident on an inhomogeneous magnetic field (i.e. the OG Stern-Gerlach experiment with spin-1/2 particles).

- (5 marks) Suppose we have measured $\langle S_x \rangle$, $\langle S_y \rangle$ and $\langle S_z \rangle$ but we do not know the initial state of the system. Is it possible to know the density matrix of the system, ρ ?
- (3 marks) Now suppose that we know that the system is pure. Is it possible to determine ρ with fewer measurements?

Exercise 2 *Laser cooling* (20 points)

- We are going to perform some calculations based on the simplest laser-cooling system, namely a laser beam incident on a source of thermal atoms.
 - Find the distance required to stop a room-temperature rubidium atom with the resonant scattering force
 - Show that rubidium atoms with velocity $v = 350$ m/s are resonant with a counter-propagating laser with a frequency detuning of 450 MHz
 - In class, we looked at using a magnetic field to deal with the changing Doppler shift of the light as seen by the atom. An alternative method is to change the laser frequency, known as *chirping* the frequency. What is the maximum rate at which one could chirp the laser frequency to cool rubidium atoms? Assuming a Doppler width of 1 GHz, how long would it take to slow the atoms?
 - In laser cooling experiments, it turns out to be necessary to have (at least) two laser beams to actually cool atoms; in the case of rubidium, only two are required. Suggest what is the reason that multiple lasers are required, and describe what properties (e.g. intensity, frequency, polarisation) that these laser beams should have, and why?
- The Magneto-Optical Trap (MOT) is the workhorse of atomic physics, having revolutionised how ensembles of atoms can be prepared and interrogated.
 - Explain what a MOT is, and how it works
 - Imagine that you have a MOT: it is completely standard except you altered the beams which propagate along the vertical axis to have a frequency difference between them; normally, all six beams have the same detuning $\Delta \approx -\Gamma/2$, but here the beams in the horizontal plane have the detuning Δ , but the vertical beams each have a detuning $\Delta \pm \delta$. What will happen?
 - Atoms trapped in a MOT are typically imaged using absorption imaging, that is, illuminating the atom cloud with an on-resonant laser beam, and then imaging the profile of the laser

beam, which will have a “shadow” where the atoms were. What effect will this imaging have on the atoms?

- iv) Assuming that the system can be described in the steady state, what would be the effect of imaging with an slightly detuned laser?

Exercise 3 Ramsey interferometry (24 points)

In this question, we are going to explore Ramsey interferometry in a two-level atom.

- a. Explain how Ramsey interferometry allows for a precise determination of the resonance frequency in a two-level system

As seen in class, the excited-state population for a general detuning Δ for Ramsey interferometry can be calculated via

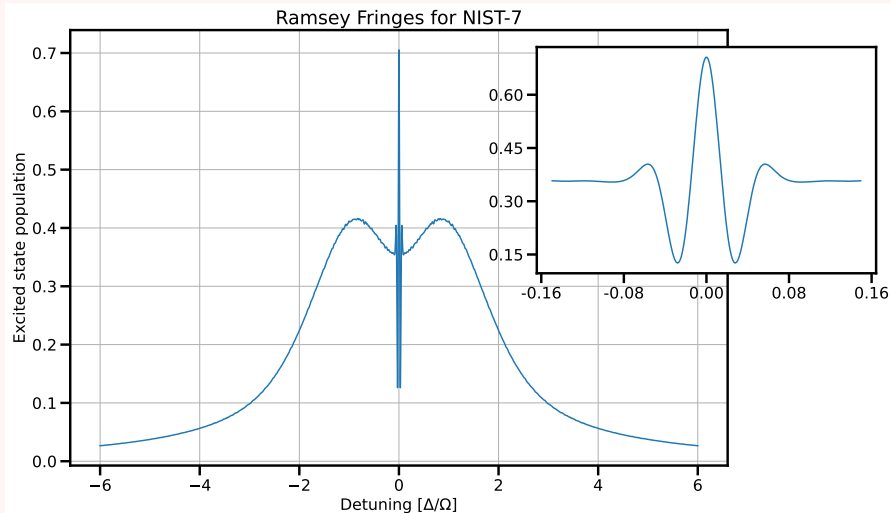
$$\rho_{22} = 4 \left(\frac{\Omega}{\Omega'} \right)^2 \sin^2 \left(\frac{\Omega' \tau}{2} \right) \left[\cos \left(\frac{\Delta T}{2} \right) \cos \left(\frac{\Omega' \tau}{2} \right) - \frac{\Delta}{\Omega'} \sin \left(\frac{\Delta T}{2} \right) \sin \left(\frac{\Omega' \tau}{2} \right) \right]^2.$$

- b. (4 marks) The NIST-7 clock as discussed in class has experimental parameters $(v, l, L, \tau, \frac{\Omega}{2\pi}) = (230 \text{ m/s}, 23 \text{ mm}, 1.53 \text{ m}, 100 \mu\text{s}, 2.5 \text{ kHz})$ for the beam velocity, interaction region width, propagation distance interaction time and Rabi frequency. Explain why these values are chosen to obtain an accurate measurement of the transition, with specific reference the the effect(s) of changing the parameters.

\begin{Computational content}

Our goal is to understand the experimental signal that is measured when the clock is operational.

- c. (3 marks) Using the experimental parameters given above, plot the expected signal - that is, excited state state population as a function of the detuning from atomic resonance, Δ
- d. (12 marks) Shown below is a (simulated) plot of the measured signal from the apparatus



This should look distinctly different to the plot you produced in the previous question. Your goal is to reproduce the above plot, which will require you to identify why the signal does not look like described by the equation above, and then you will have to simulate what is actually measured, and hopefully it will look like the above plot (which is indeed what is actually measured).

HINT: The [Maxwell-Boltzmann distribution](#) may or may not be relevant, along with the information that the temperature of the caesium beam oven was about 55 degrees C.